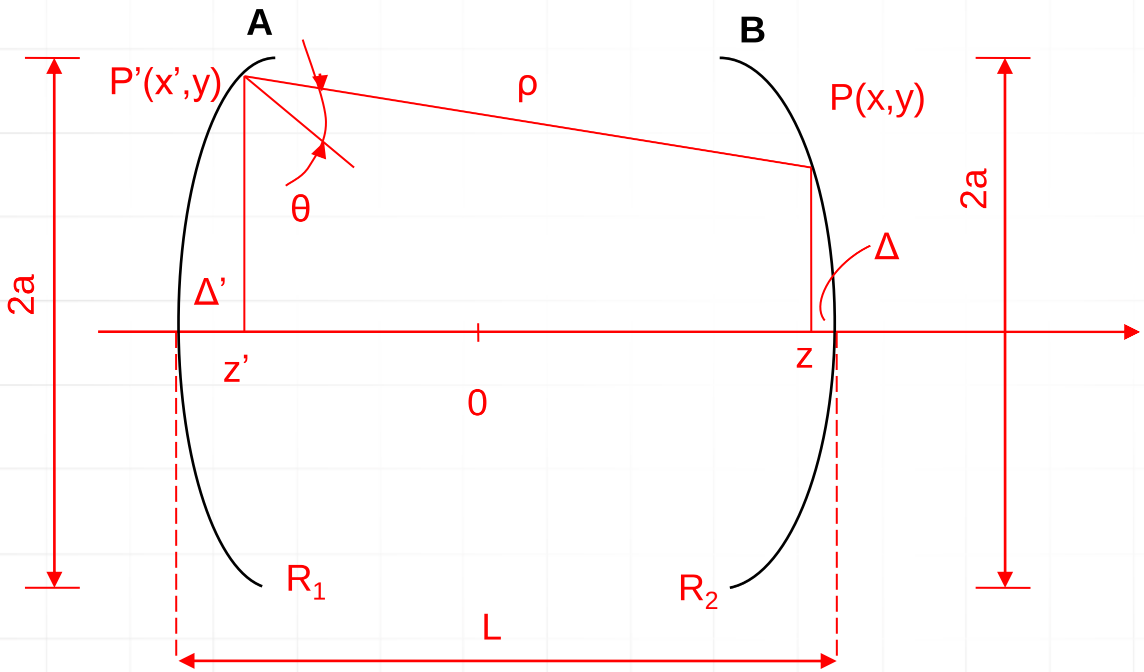
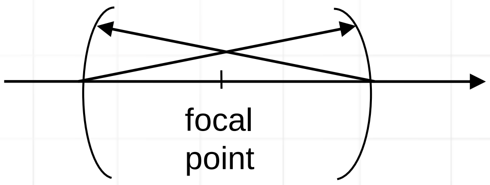
Optical resonators
Mode structure
Gaussian beams

Confocal resonator

Fabry-Perot resonator,
two spherical mirrors separated by $R_1 = R_2 = R = L$

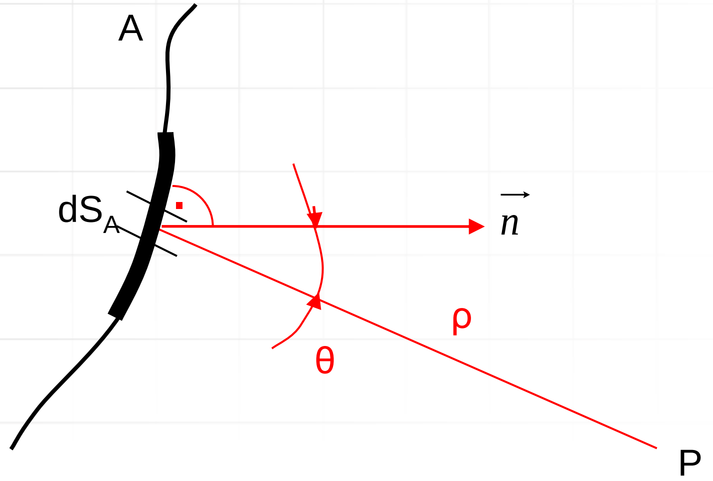


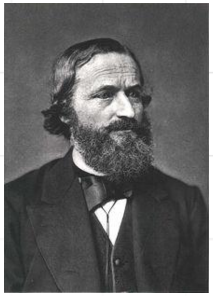


Confocal resonator – assumptions

1. Confocal cavity with spherical mirrors and square aperture (2a x 2a),
2. Both mirrors have the same radius of curvature $R_1 = R_2 = R$
3. Confocality means $R_1 = R_2 = R = L$ (focal point is in the middle of the cavity, L/2 or R/2)
4. The aperture of both mirrors (2a x 2a) and its length L, are much larger then considered wavelength $L >> a >> \lambda$
5. The cavity is empty (no gain medium inside)
6. We assume that the polarization of the electromagnetic wave inside the resonator is linearly polarized and it doesn't change during propagation
7. We assume a stationary field distribution. It means that the field distribution at both mirrors differ only by constant coefficient
8. The field at one mirror is expressed by the field at second one according to the Fresnel-Kirchoff diffraction equation which leads to an integral equation, and we will try to find so called **EIGEN FUNCTIONS** and **EIGEN VALUES**.

Fresnel-Kirchoff equation





$$U_p(P) = \frac{ik}{4\pi}\int_A U(x,y,z)\frac{e^{-ik\rho}}{\rho}\cdot(1+\cos\theta)dS_A$$

U(x,y,z) - field distribution at the surface A

(it can be electric field distribution E(x,y,z)

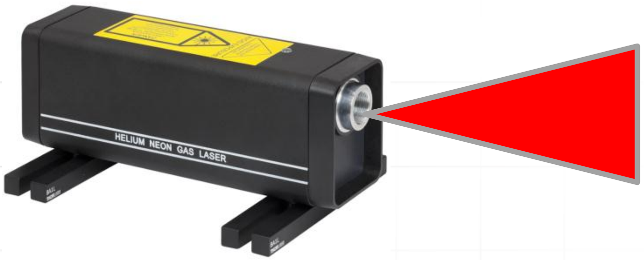
$k = \frac{2\pi}{\lambda}$ - propagation constant

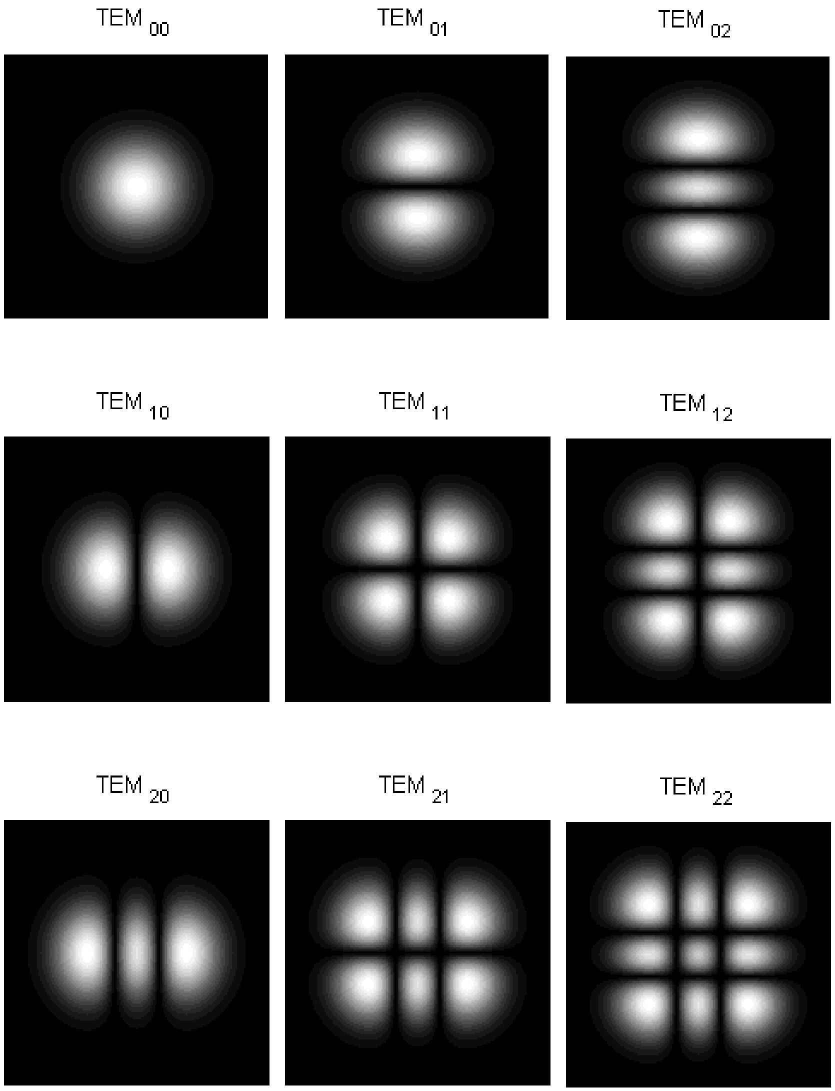
ρ - distance between elementary dS_A and point P

θ - the angle between normal to the $\overrightarrow{dS_A}$ and $\vec{\rho}$.

Solutions

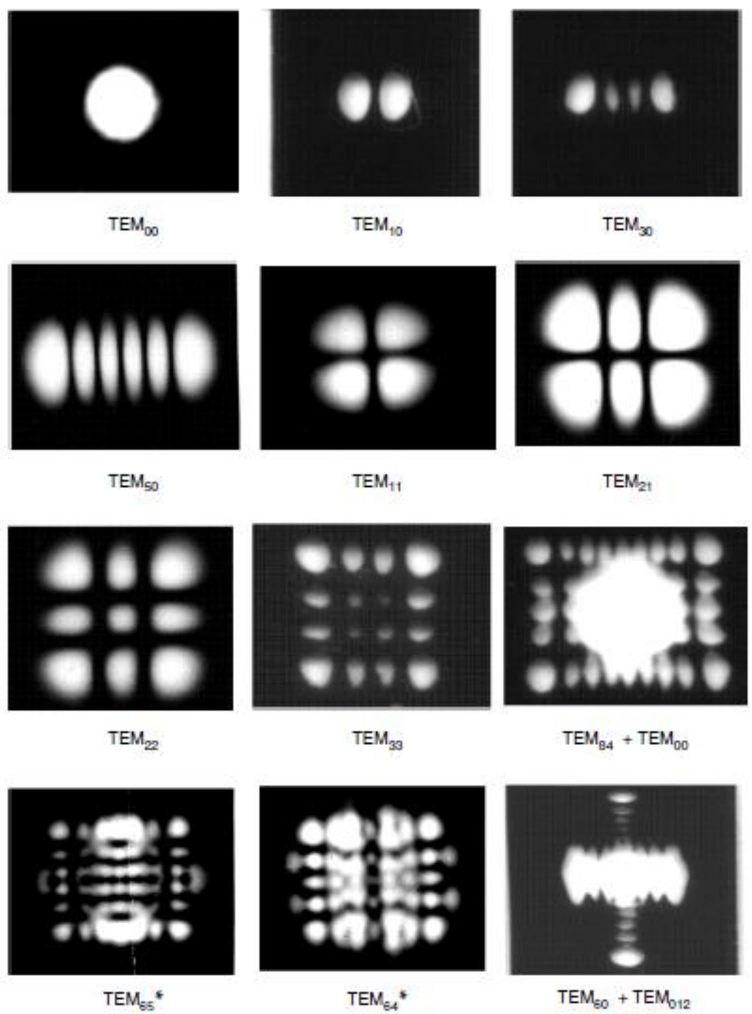
Transverse modes





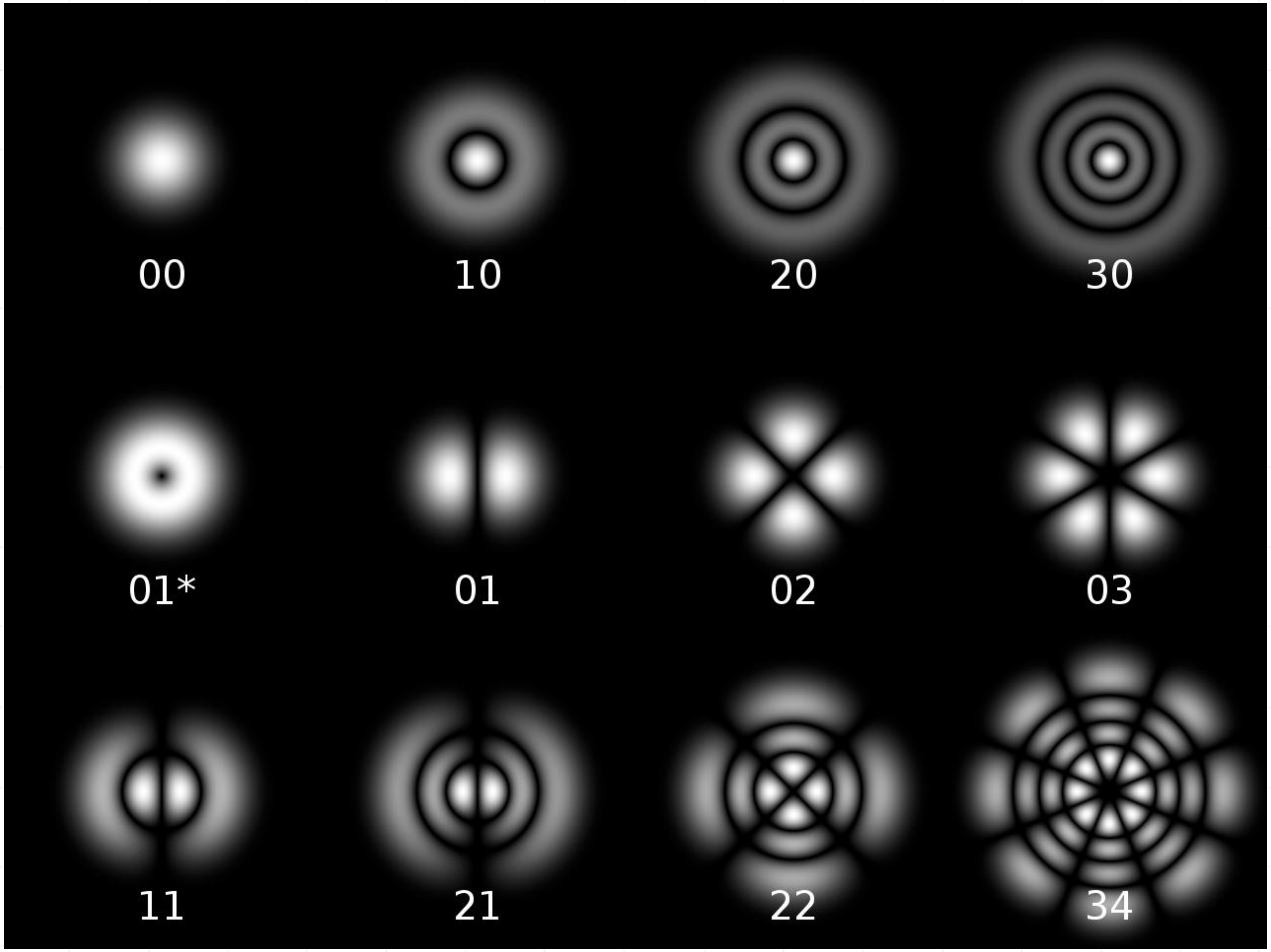
Real images (transverse modes)

$TEM_{x,y}$



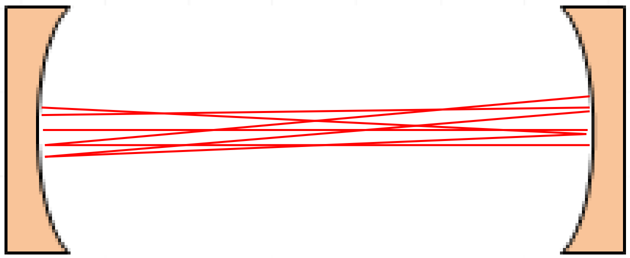
Taken from [1], K.M. Abramski, E.F. Plinski,
(M. Endo, R.F.Walter, Editors),
Gas Lasers, chapter 1, CRC Press, London, 2007

More complicated structures ....

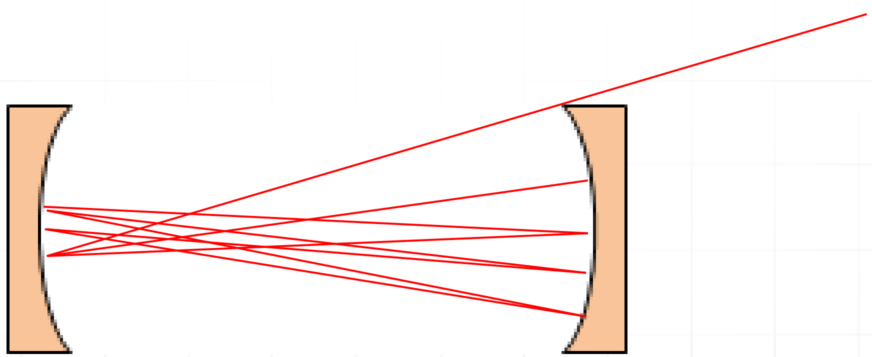


Stability of a resonator

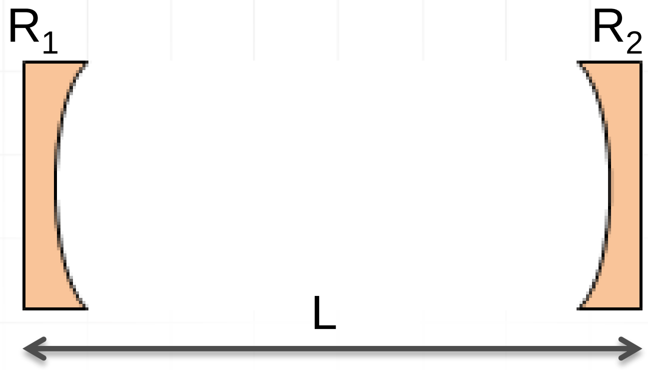
Stable



Unstable



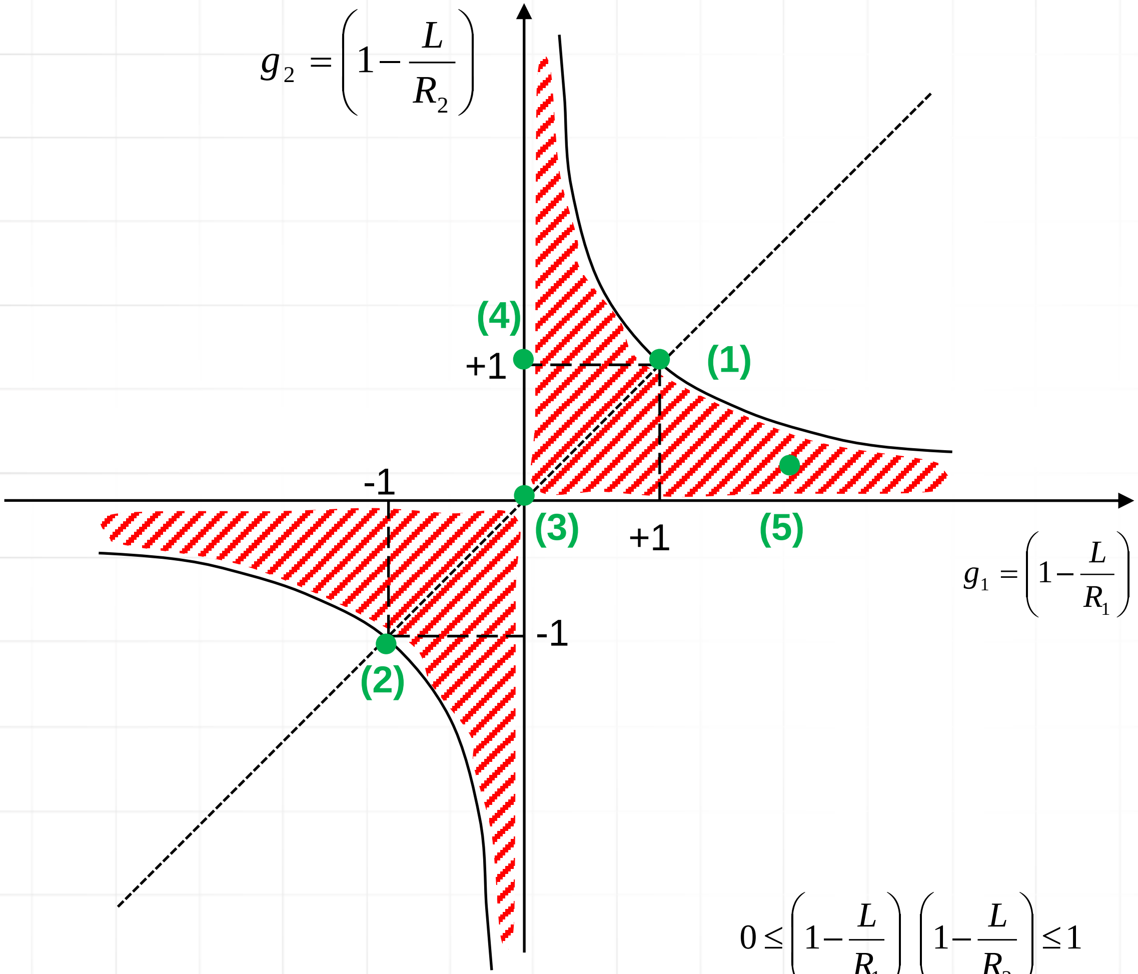
Stability conditions



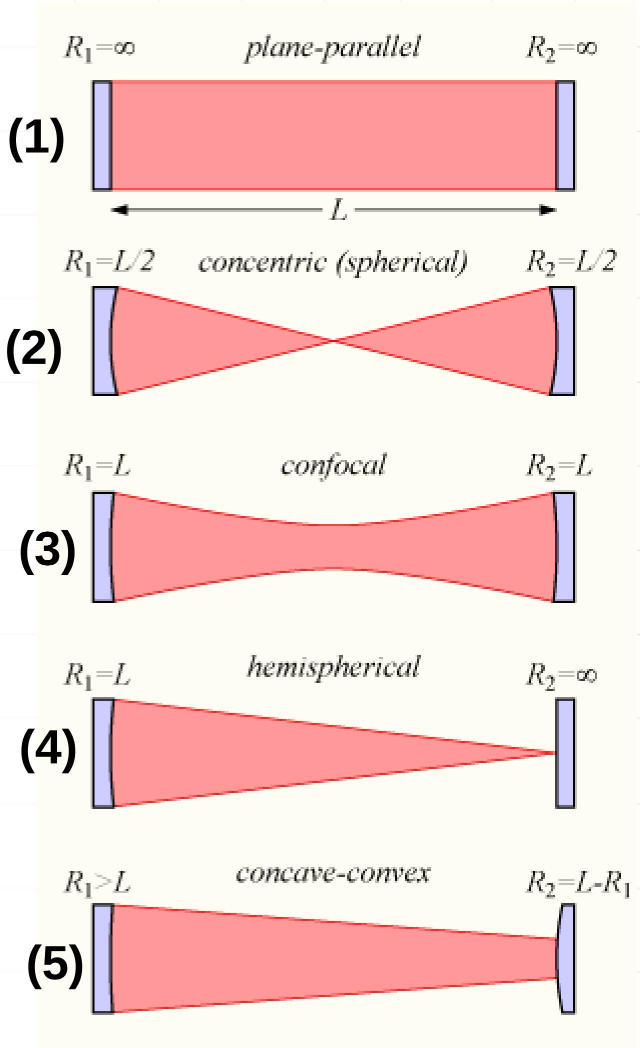
$$0 \le \underbrace{\left(1-\frac{L}{R_1}\right)}_{g_1} \underbrace{\left(1-\frac{L}{R_2}\right)}_{g_2} \le 1$$

g_1, g_2 - stability parameters of spherical resonators.

Stability diagram



$$0 \leq \left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) \leq 1$$

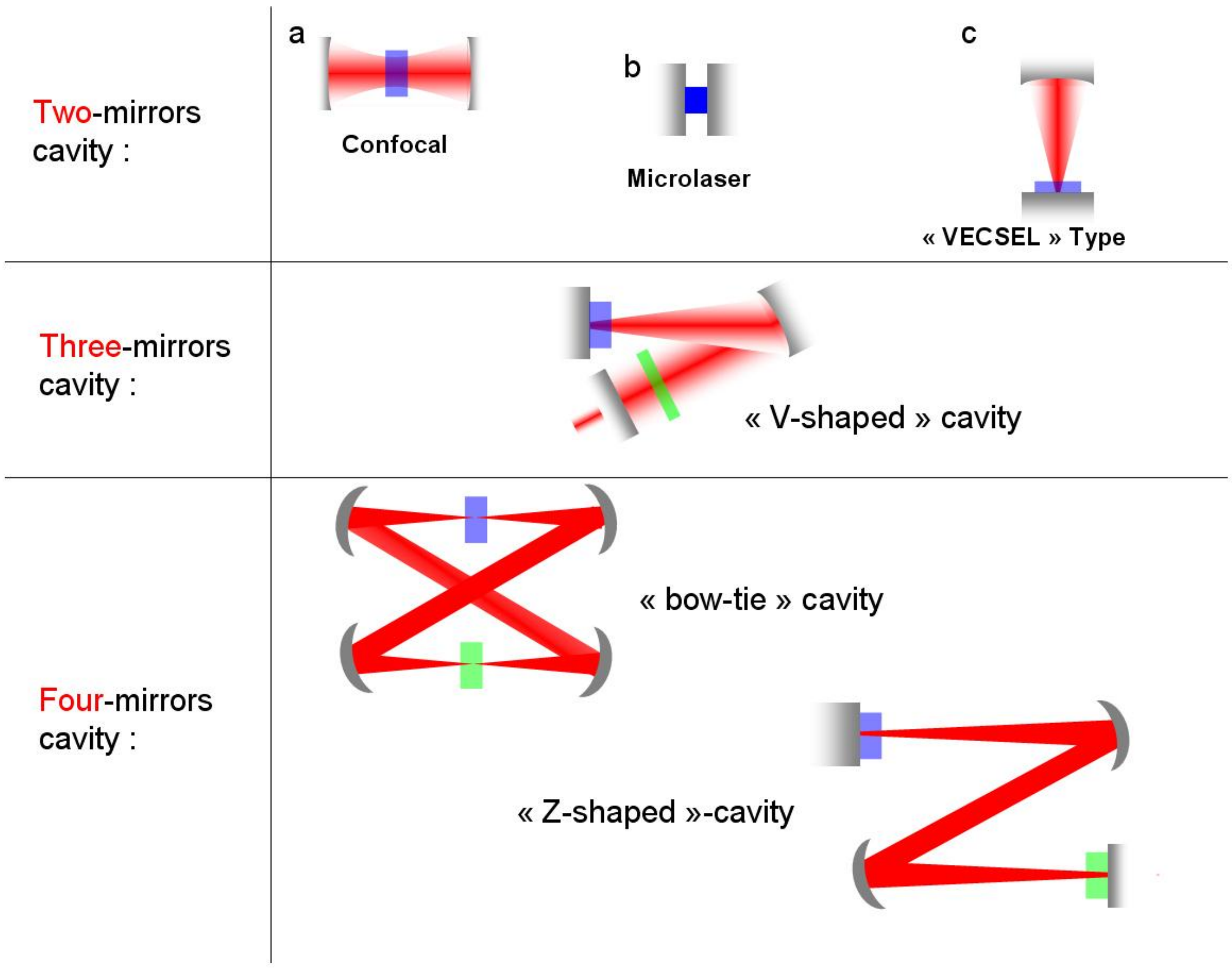


Examples

Characteristic Gaussian resonators

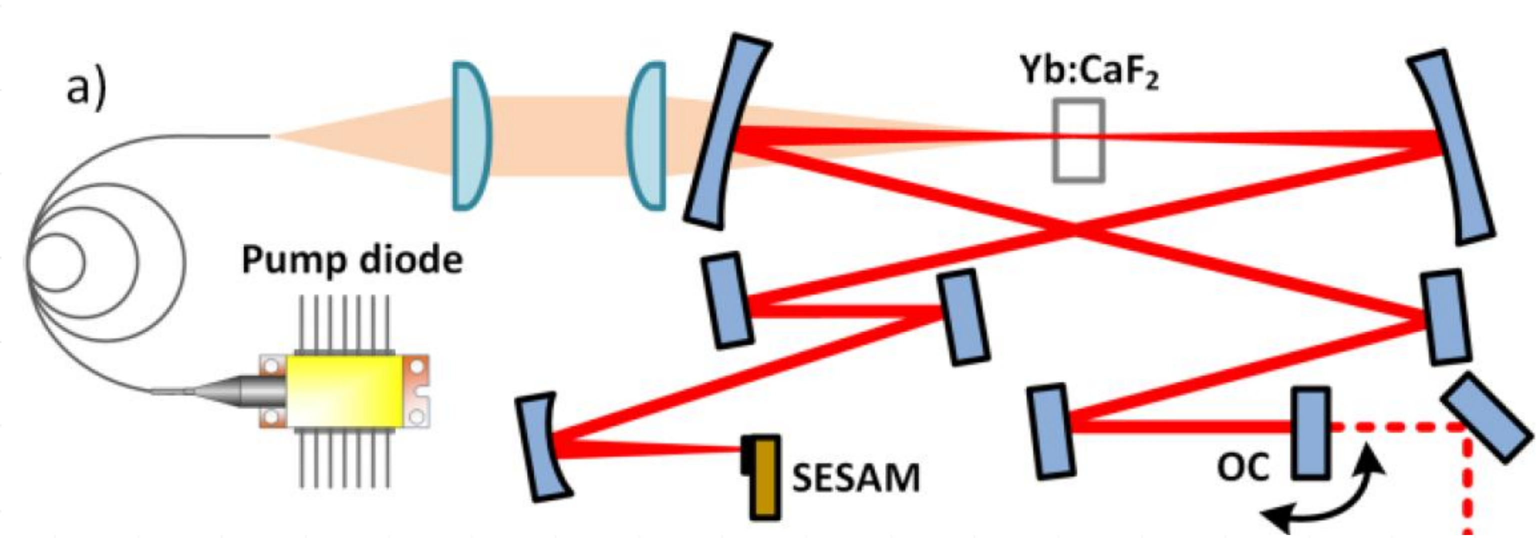
Type	**R_1**	**R_2**	**g_1**	**g_2**
plane-parallel	∞	∞	1	1
symmetric	$R_1=R_2$	$R_1=R_2$	0	0
confocal	$R_1=L$	$R_2=L$	-1	-1
concentric	$R_1=L/2$	$R_2=L/2$	1	1/2
halfconfocal	∞	$R_2=2L$	1	1/2
halfconcentric	∞	l	1	0
Concave-convex	> L	$L-R_1$	2	1/3
Hemispherical	L	∞	0	1

Other resonators



Other resonators

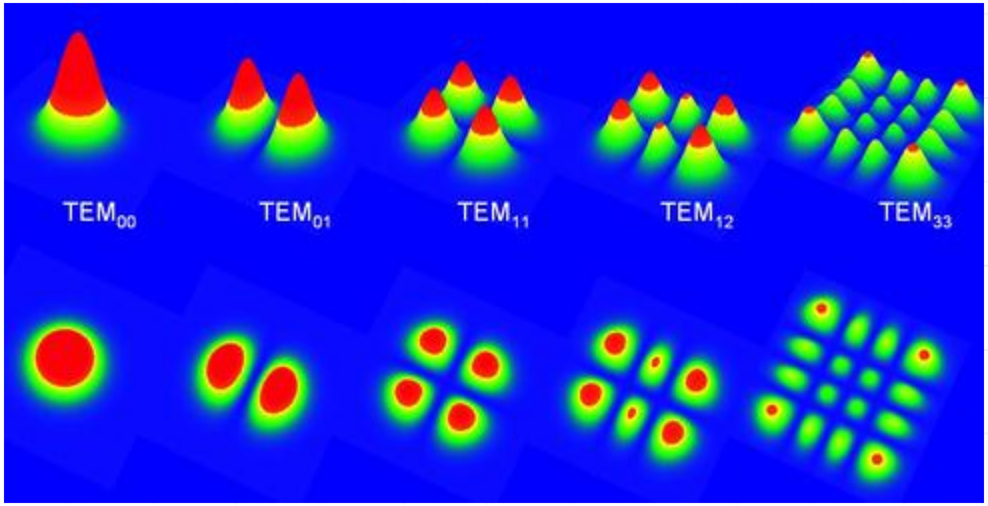
- Bow-tie laser



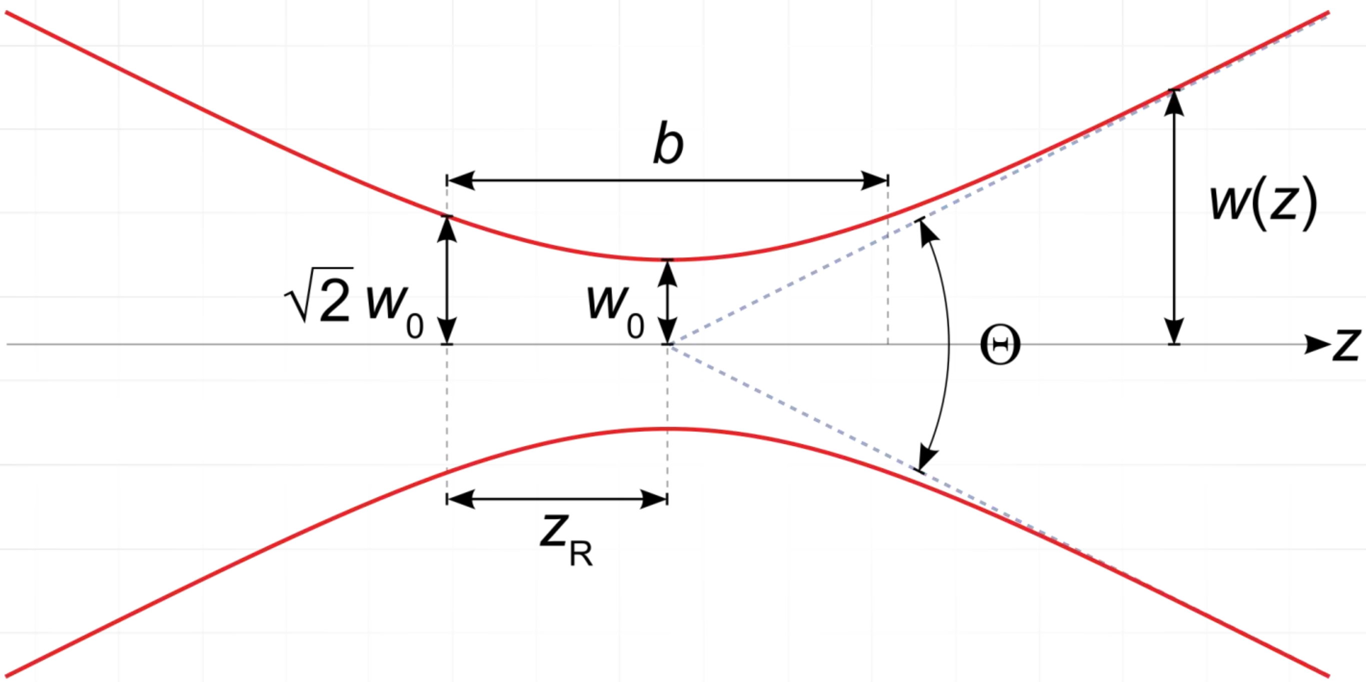
M. Kowalczyk / PWr

GAUSSIAN BEAMS

Carl Friedrich Gauss



Gaussian beam (TEM_{00})



w_0 – beam waist (radius in the smallest point)
w*(z)** – radius of the Gaussian beam at position ***z
R(z) – radius of the wave front
Θ – divergence angle of the beam
Z_R – Rayleigh range

Parameters of a Gaussian beam

Radius of the beam at point **z**
(spot size)

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}$$

Rayleigh range

$$z_R = \frac{\pi w_0^{\ 2}}{\lambda}$$

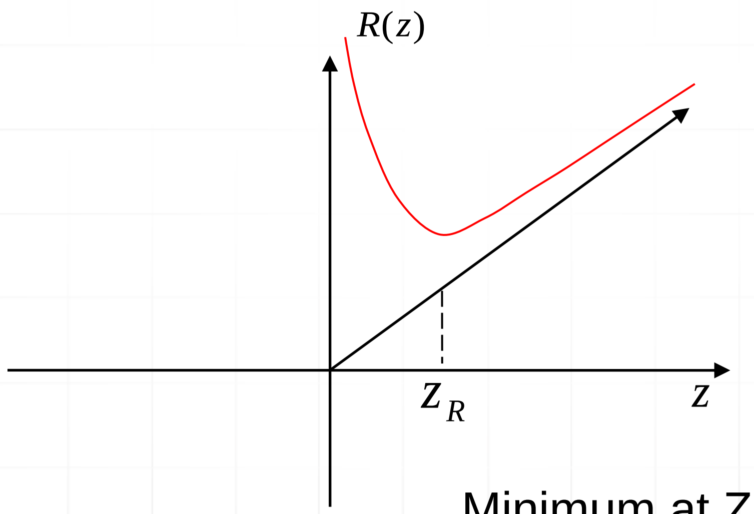
If z = z_0, then:

$$w(z) = \sqrt{2} w_0$$

Parameters of a Gaussian beam

Radius of curvature

$$R(z) = z\left[1+\left(\frac{z_R}{z}\right)^2\right]$$

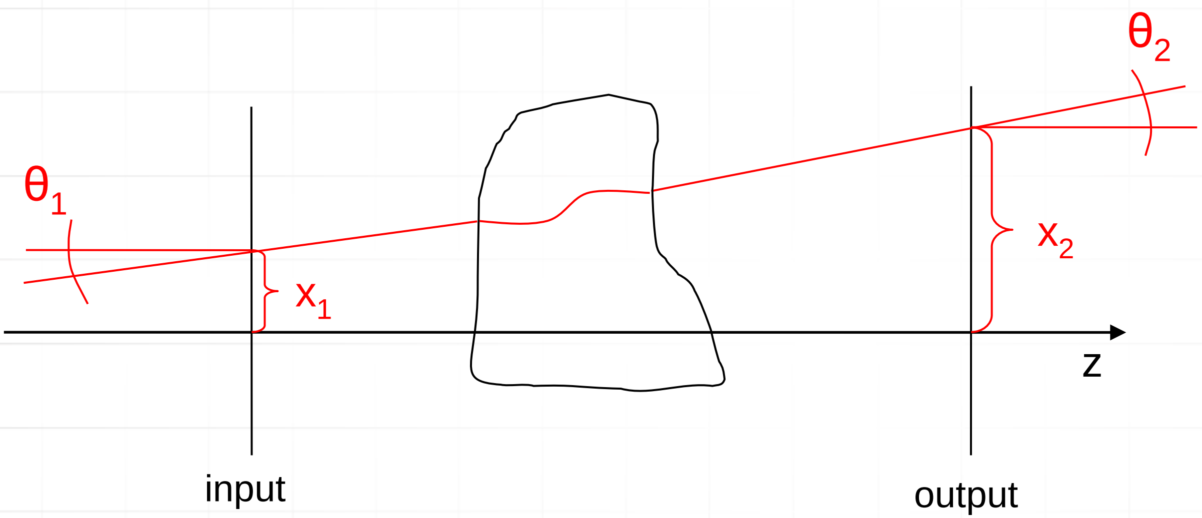


Minimum at Z_R

At z=0, the radius is infinitely big (plane wave)

Ray transfer matrix analysis

- ABCD matrix



$$\begin{bmatrix} x_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix}$$

In ray transfer ABCD matrix analysis, an optical element gives a transformation between (x_1,θ_1) at the input plane and (x_2,θ_2) when the ray arrives at the output plane.

Examples of ABCD matrices

Propagation in free space through distance ***d***

$$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Reflection from a flat mirror

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thin lens

$$\begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$$

Gaussian beams

For a spherical wave:

$$x = R\theta$$

$$\begin{bmatrix} R_2\theta_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} R_1\theta_1 \\ \theta_1 \end{bmatrix}$$

(1) $$R_2\theta_2 = AR_1\theta_1 + B\theta_1$$

(2) $$\theta_2 = CR_1\theta_1 + D\theta_1$$

hence

$$R_2 = \frac{AR_1 + B}{CR_1 + D}$$

Gaussian beams

Complex beam parameter of a Gaussian beam (q) :

$$\frac{1}{q} = \frac{1}{R} - i\,\frac{\lambda}{\pi w^2}$$

Because Gaussian beams have divergence and radius of curvature

When the beam propagates in one direction (z), and the waist is at z_0

$$q = (z - z_0) + iz_R$$

$$\begin{bmatrix} q_2 \\ 1 \end{bmatrix} = k \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} q_1 \\ \theta_1 \end{bmatrix}$$

$$q_2 = k(Aq_1 + B)$$

$$1 = k(Cq_1 + D)$$

Therefore, ABCD transformation of a Gaussian beam:

$$q_2 = \frac{Aq_1 + B}{C\,q_1 + D}$$

Gaussian beams

- Example: free space

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

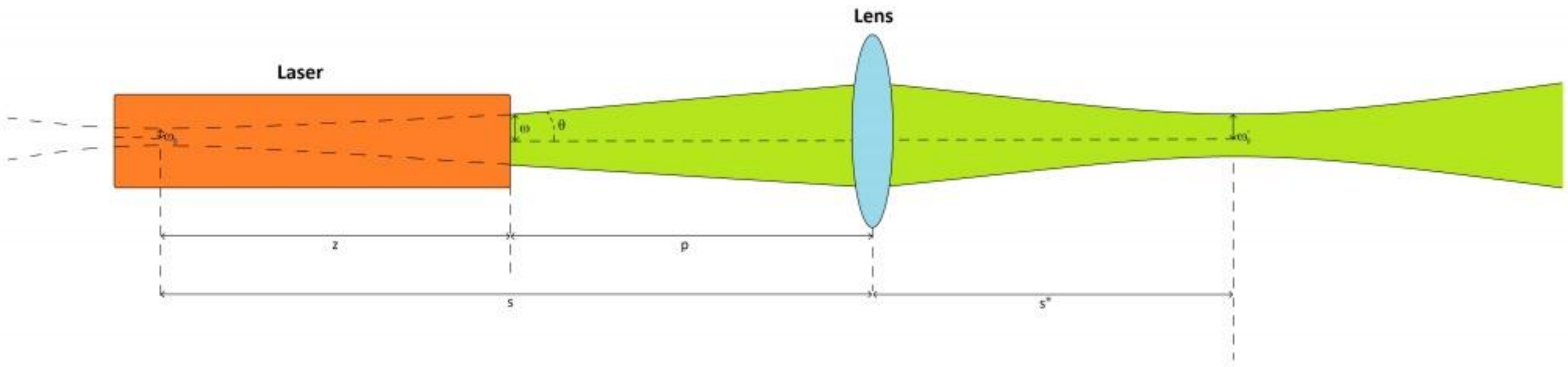
So:

$$q_2 = \frac{A q_1 + B}{C q_1 + D} = \frac{q_1 + d}{1} = q_1 + d$$

$$q = (z - z_0) + i z_R$$

As the beam propagates, both the radius and waist change.

Example: lens



Important:

- Stability map of resonators
- Gaussian beam: shape and parameters