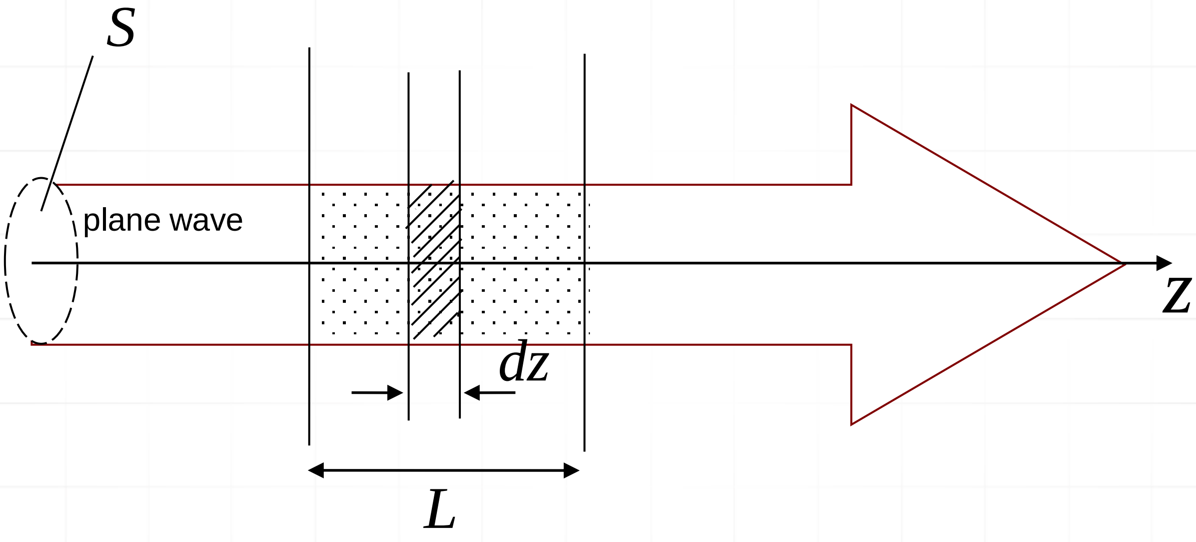
Quantum amplification

Lets assume a plane wave passing through a gain medium (L – thickness)
We can write the equations describing the emission and absorption (based on Einstein's transitions)



$$\begin{cases} dN_{em} = N_2 B_{21} u_\upsilon \, dt + N_2 A_{21} dt \\ dN_{abs} = N_1 B_{12} u_\upsilon \, dt \end{cases}$$

Quantum amplification: gain

$$g = \frac{1}{I}\frac{dI_{\upsilon_0}}{dz}$$

Differential gain

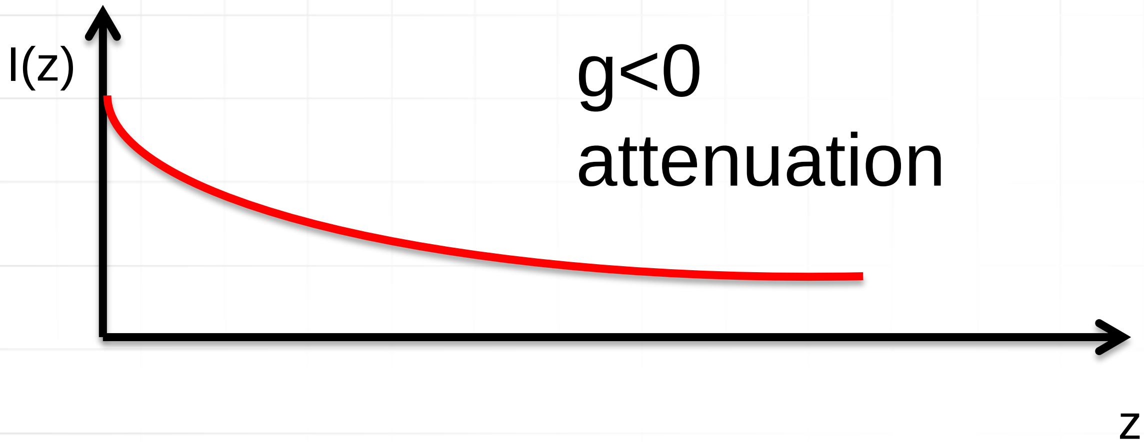
when g>0 – the wave is amplified
when g<0 – the wave is absorbed

$$g = \left(N_2 - N_1\right)\frac{Bh\upsilon}{c}$$

N_1 – population of the lower energy level
N_2 – population of the upper energy level
B – gain coefficient

If N_1 is larger than N_2 – the gain is always negative (attenuation)

Quantum amplification: gain



$$I(z) = I_0 e^{gz}$$

Problem:

From the Boltzman distribution →

in thermal equilibrium, N_2 is always <u>smaller</u> than N_1

$$N_2 < N_1 \rightarrow g < 0$$

If the medium is in thermal equilibrium, there is always only attenuation

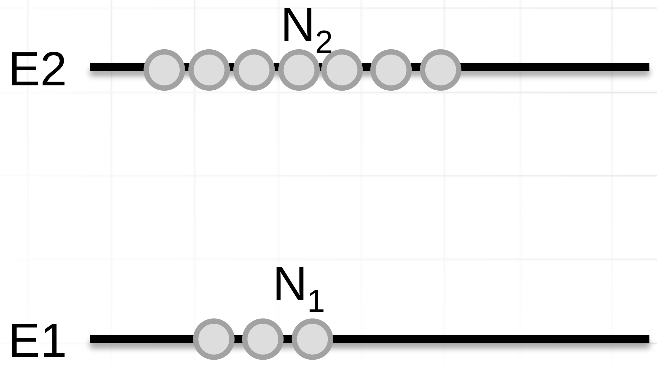
Amplification

- However, if: $N_2 > N_1$ ➔ **population inversion**

$$I(z) = I_0 e^{gz}$$

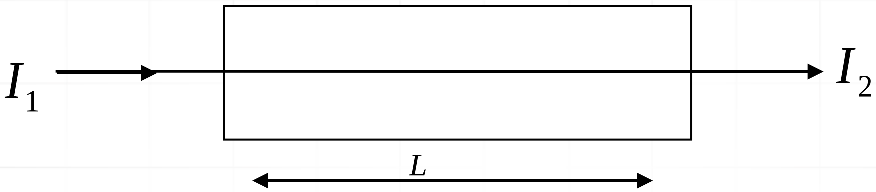
Population inversion

- $N_2 > N_1$
- $g > 0$



- How to achieve population inversion?
  - Pumping (in solid-state or fiber lasers)
  - Electric discharge (in gas lasers)
  - Electric field (in semiconductor lasers)

Amplification

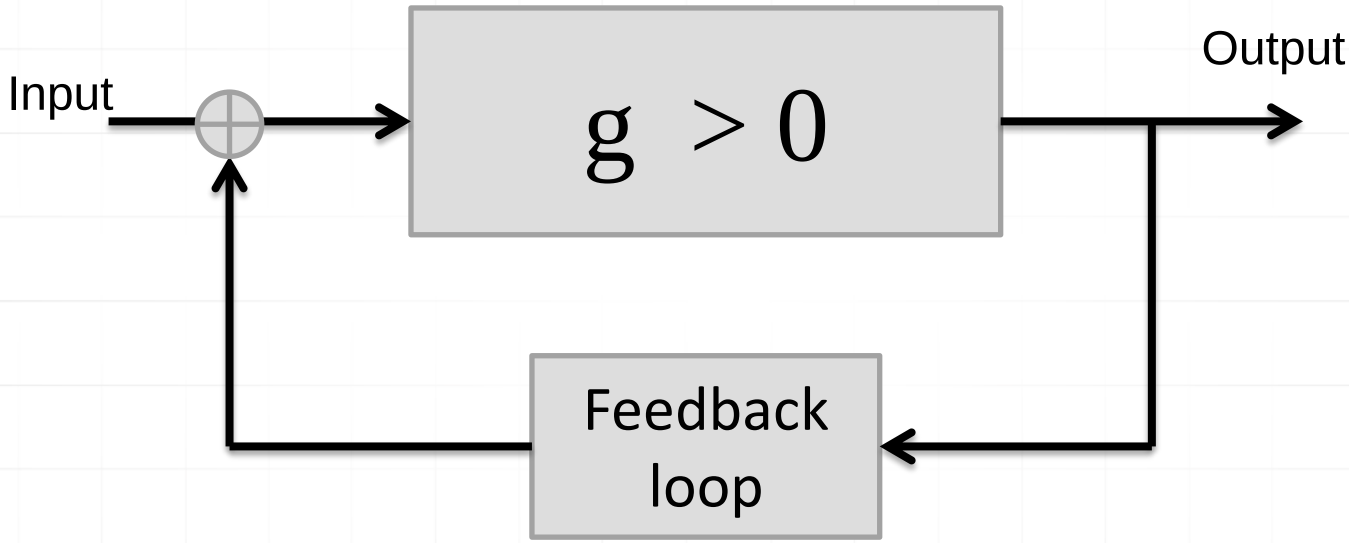


$$\frac{I_2}{I_1} = G_0 \qquad I_2 = I_1 e^{gL}$$

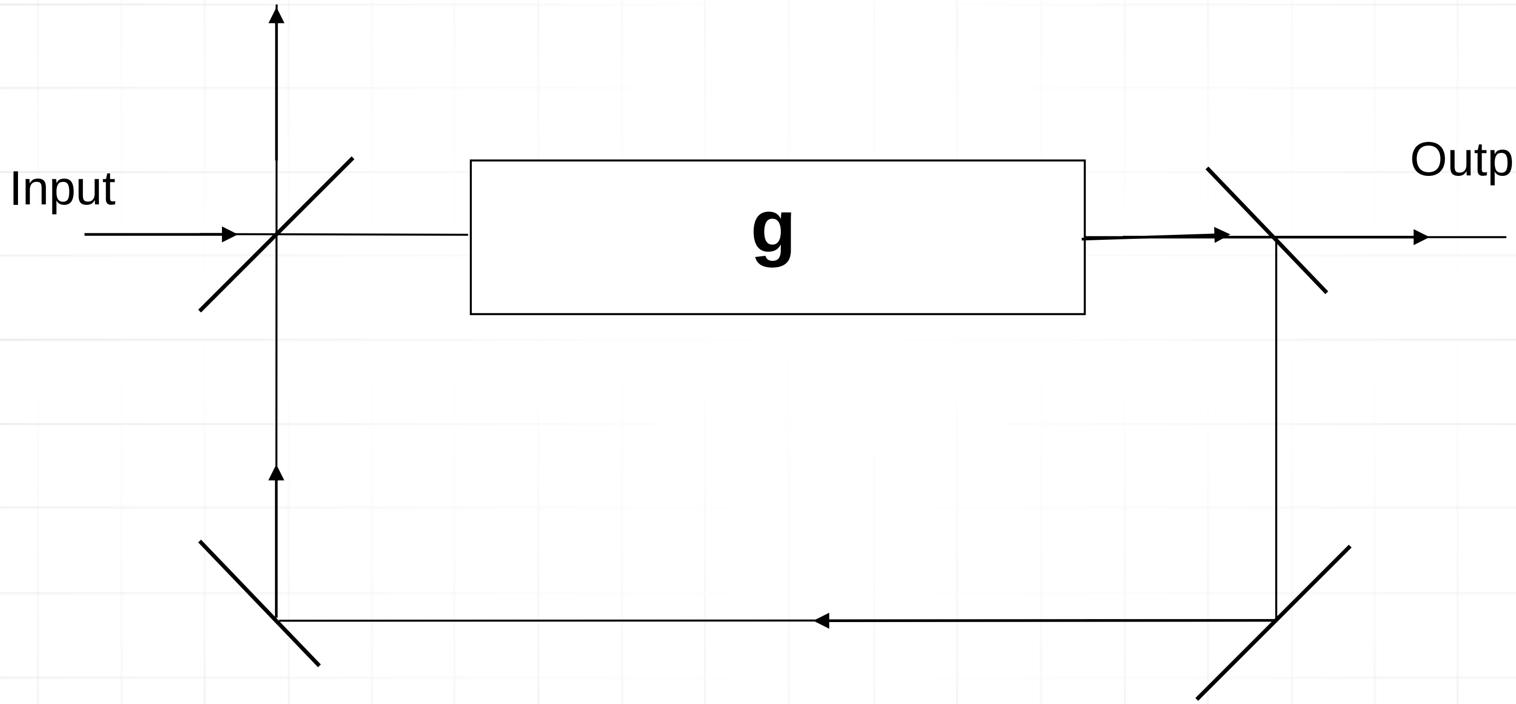
$$G_{dB} = 10 \log \frac{I_2}{I_1}$$

$$\frac{I_2}{I_1} = 10^{\frac{G}{10}}$$

How to make a laser from an amplifier?



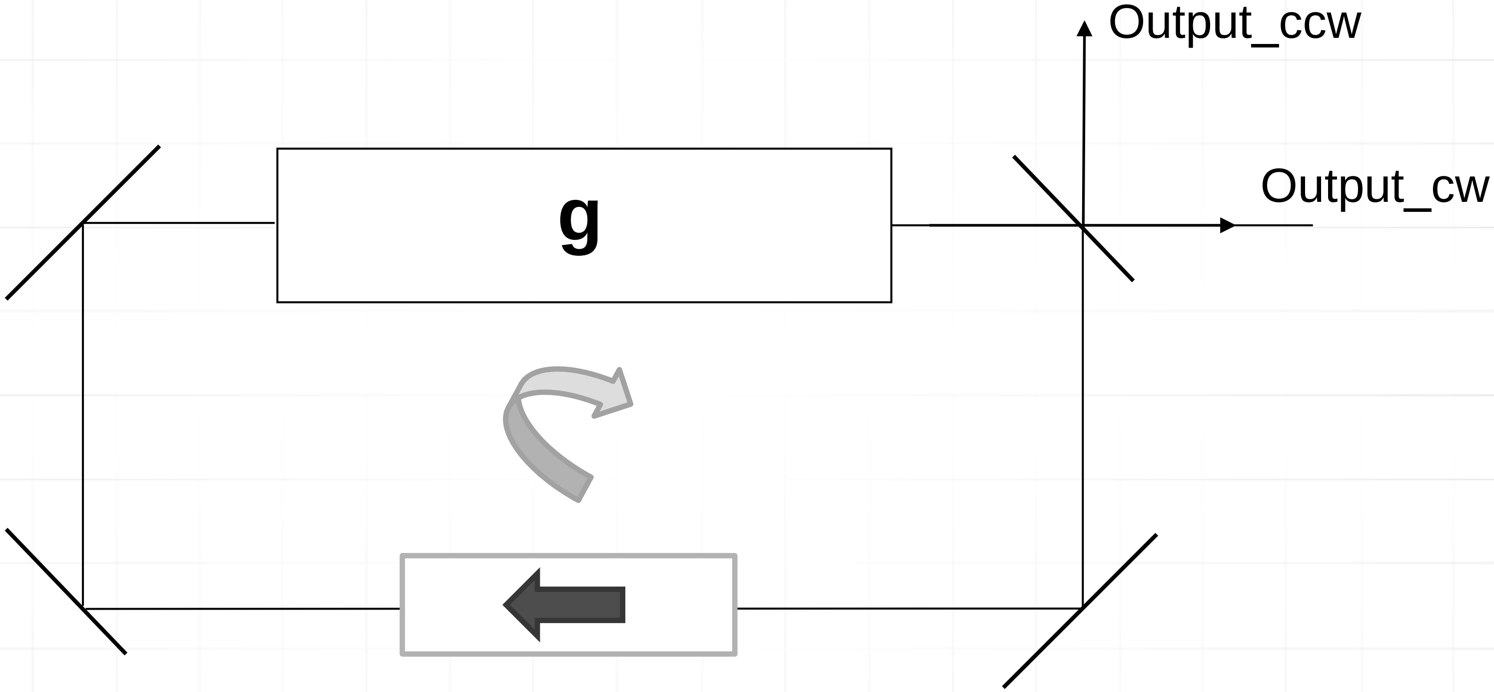
Quantum amplifier / generator



Ring resonator

Quantum amplifier / generator

Let's remove the input beam

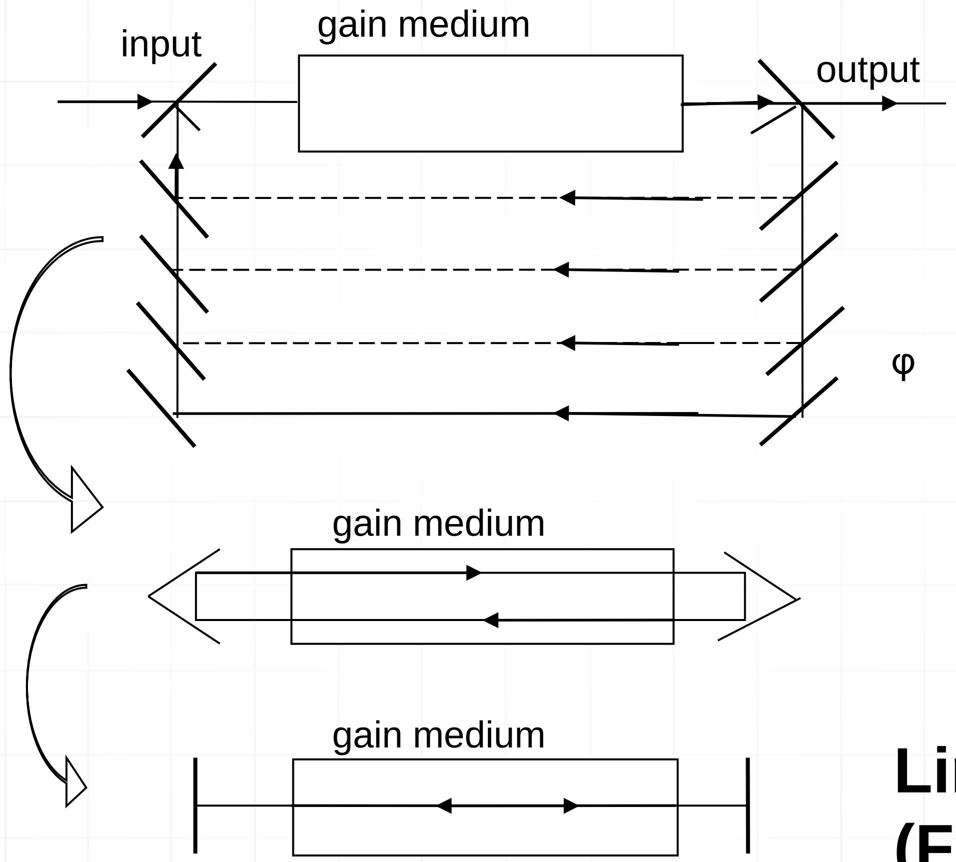


The radiation might circulate in both directions

Isolator → unidirectional operation of the laser

Ring resonator

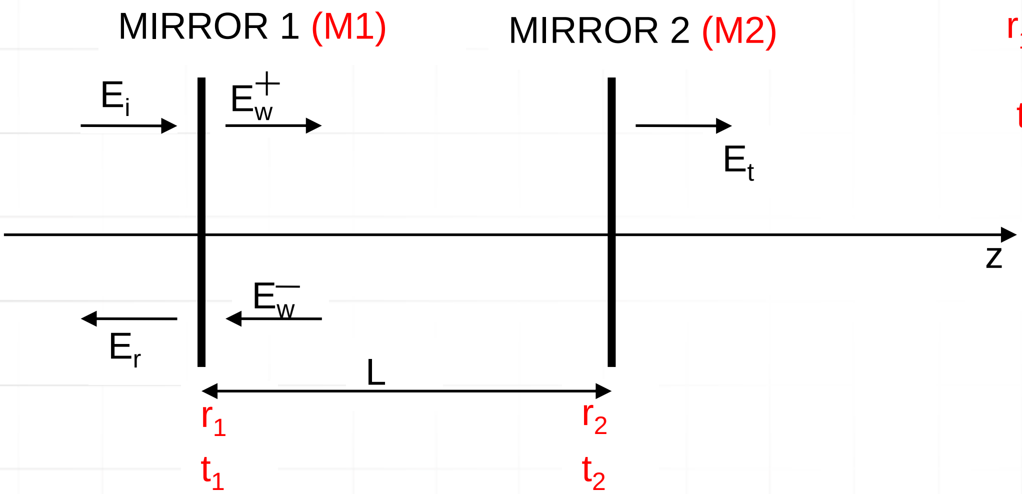
Quantum amplifier / generator



Linear resonator (Fabry – Perot)

Fabry-Pérot resonator

Two pararell mirrors separated by a distance „*L*"



r_1, r_2 - amplitude reflectivity of mirrors M1 and M2

t_1, t_2 - amplitude transmission of mirrors M1 and M2

$$|r|^2 = R \qquad |t|^2 = T$$

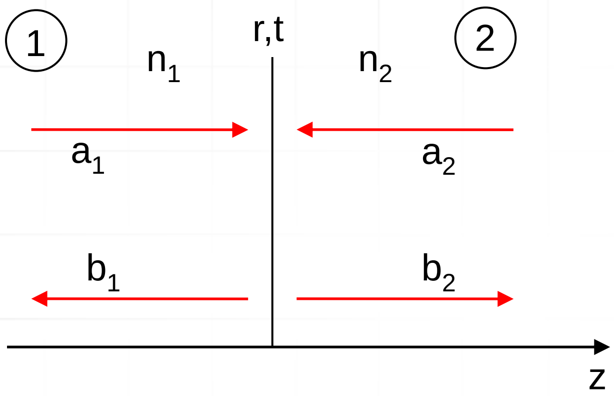
$$|r|^2 + |t|^2 = 1 \qquad R + T = 1$$

Charles Fabry
1867-1945

Alfred Pérot
1863-1925

Fabry-Pérot resonator

Let consider the perpendicular transmission of a plane wave through the border of two dielectrics. It is represented in the figure below:



$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

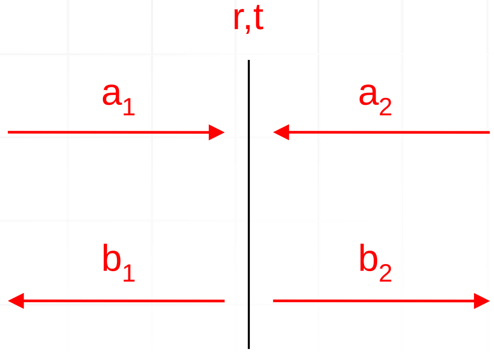
$$t = \frac{2\sqrt{n_1 n_2}}{n_1 + n_2}$$

$$E_1(z,t) = a_1 \exp[i(\omega t + k_1 z)] + b_1 \exp[i(\omega t - k_1 z)]$$

$$E_2(z,t) = a_2 \exp[i(\omega t + k_2 z)] + b_2 \exp[i(\omega t - k_2 z)]$$

Fabry-Pérot resonator

From the figure



We can define a set of equations:

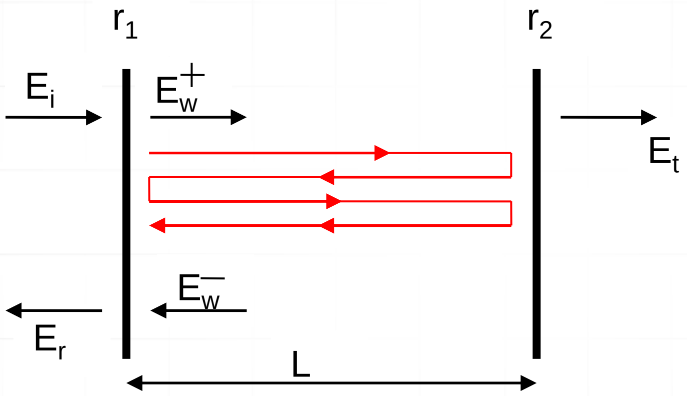
$$\begin{cases} b_1 = ra_1 + ta_2 \\ b_2 = ta_1 - ra_2 \end{cases}$$

or the matrix equation: (so-called SCATTERING MATRIX)

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} r & t \\ t & -r \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Fabry-Pérot resonator

Lets calculate the transmission and reflection of a F-P resonator



$$E_w^{\ +} = \sum_{0}^{\infty} E_n^{\ +}$$

Fabry-Pérot resonator

Transmission coefficient:

$$T = \left|\frac{E_t}{E_i}\right|^2 = \frac{(1-R_1)(1-R_2)}{\left(1-\sqrt{R_1R_2}\right)^2 + 4\sqrt{R_1R_2}\sin^2\theta}$$

Reflection coefficient

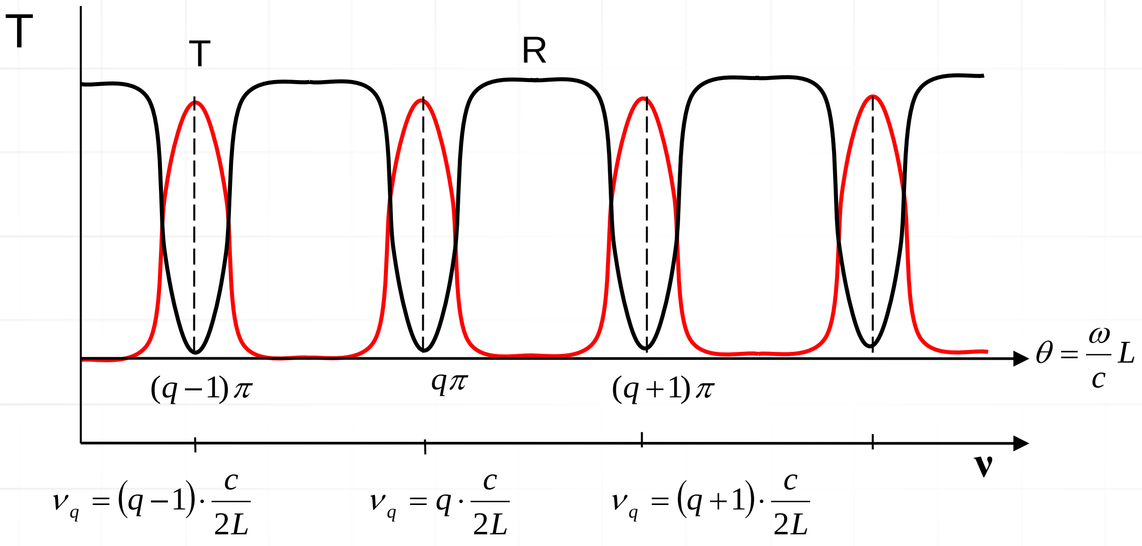
$$R = \left|\frac{E_r}{E_i}\right|^2 = \frac{\left(\sqrt{R_1}-\sqrt{R_2}\right)^2 + 4\sqrt{R_1R_2}\sin^2\theta}{\left(1-\sqrt{R_1R_2}\right)^2 + 4\sqrt{R_1R_2}\sin^2\theta}$$

From above equations we have:

$$R + T = 1$$

Introducing

$$\theta = \frac{\omega}{c} L$$

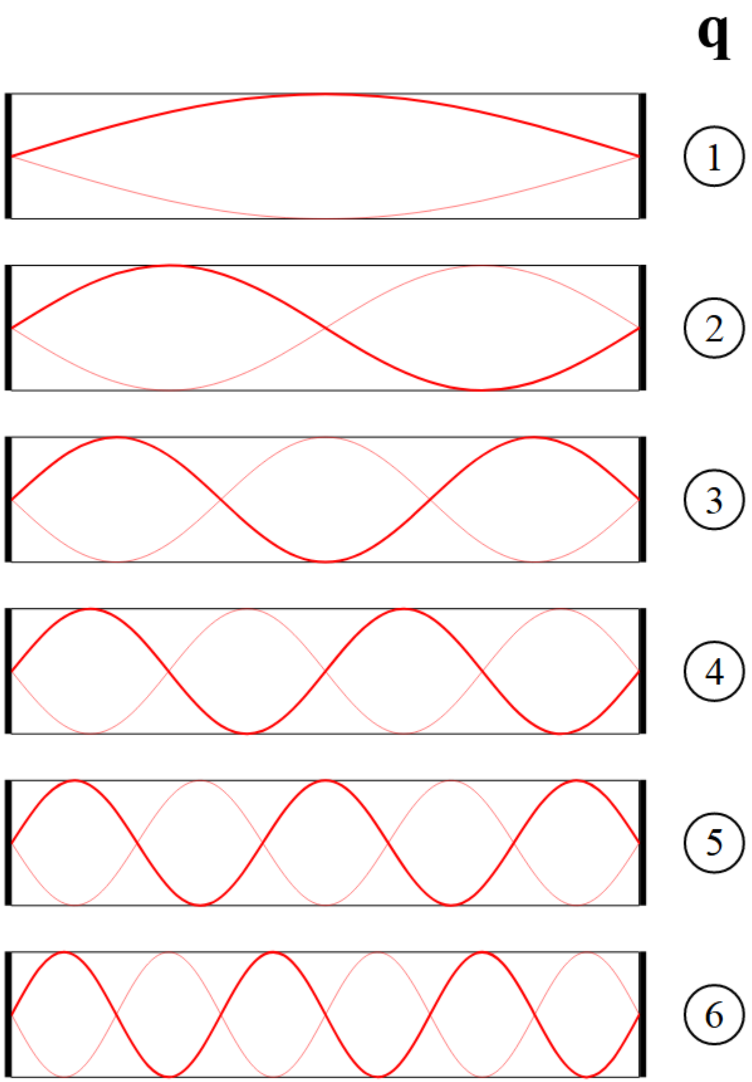


We have maxima of transmission for cases:

$$\theta = q\pi \quad , \quad \frac{\omega L}{c} = q\cdot\pi \quad , \quad \frac{2\pi\nu_q}{c} = q\cdot\pi$$

$$\boxed{\nu_q = q\cdot\frac{c}{2L}}$$

Resonances



The waves need to „fit" into the length of the resonator

$$L = q\frac{\lambda}{2}$$

but:

$$\lambda = \frac{c}{\nu}$$

Hence:

$$\nu_q = q \cdot \frac{c}{2L} \quad , \quad \nu_{q+1} = (q+1) \cdot \frac{c}{2L}$$

Free-spectral range (FSR)

- Difference between two consecutive resonances

(1)

$$\Delta\nu_{FSR} = \nu_{q+1} - \nu_q = \frac{c}{2L}$$

$\Delta\nu_{FSR}$ - depends only on optical length of resonator

For example:

$$L = 1m \quad , \quad \Delta\nu_{FSR} = 150\,MHz$$

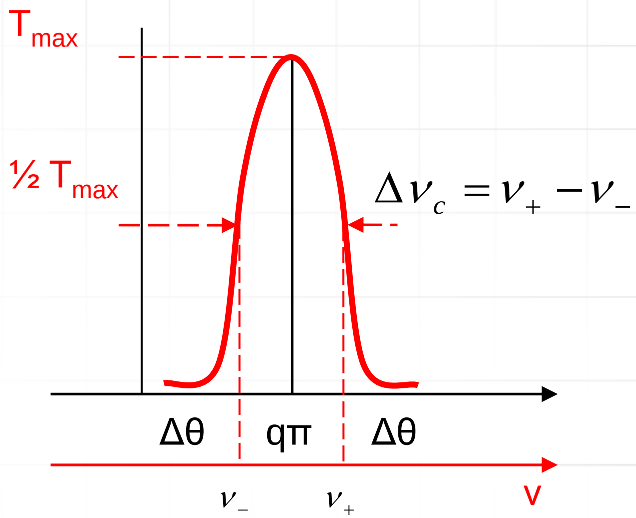
$$L = 1cm \quad , \quad \Delta\nu_{FSR} = 15\,GHz$$

$$L = 1mm \quad , \quad \Delta\nu_{FSR} = 150\,GHz$$

Linewidth of the resonance

Lets find the width of the resonance:

$$T_{max} = \frac{(1-R_1)(1-R_2)}{\left(1-\sqrt{R_1R_2}\right)^2}$$



Let assume:

$$R_1 = R_2 = R$$

$$T_{max} = \frac{(1-R)^2}{(1-R)^2} = 1$$

$$T_{min} = \frac{(1-R)^2}{(1+R)^2}$$

(2)

$$\Delta\nu_c = \frac{c}{2L}\frac{1-\sqrt{R_1R_2}}{\pi(R_1R_2)^{1/4}}$$

so-called “empty cavity linewidth”

Q-factor

Other important parameters of the Fabry-Perot cavity:

Q- factor

(3)

$$Q = \frac{\nu_0}{\Delta \nu_c}$$

$$Q = \frac{2\pi L}{\lambda_0} \frac{(R_1 R_2)^{1/4}}{1 - (R_1 R_2)^{1/2}}$$

For example:

$R_1 = 1.0$, $R_2 = 0.99$

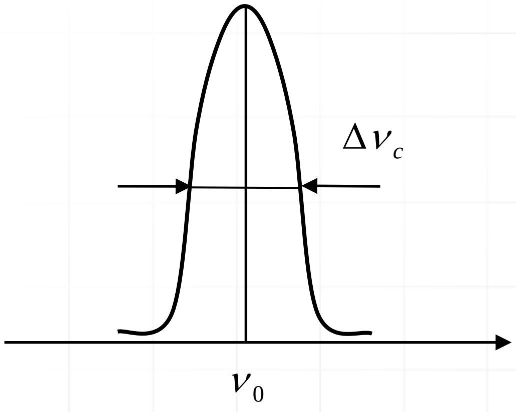
$L = 1$ m

$\lambda = 632{,}8$ nm

$v = 4.74 \cdot 10^{14}$ Hz

$Q = 9.88 \cdot 10^{8} = 10^{9}$

$\Delta v_c = 480$ kHz $= 0{,}5$ MHz



Finesse

The next important parameter – “Finesse”

$$\Im = \frac{free\ spectral\ range}{\Delta\nu_c} = \frac{\Delta\nu_{FSR}}{\Delta\nu_c}$$

The third parameter of empty cavity:

(4)

$$\Im = \frac{\pi\left(R_1 R_2\right)^{1/4}}{1-\left(R_1 R_2\right)^{1/2}}$$

Typical value:

$$\Delta\nu_{FSR} = 150\,MHz$$

$$\Delta\nu_c = 0.5\,MHz$$

$$\Im = 300$$

Summary - parameters

1) Free spectral range $$\Delta\nu_{FSR} = \nu_{q+1} - \nu_q = \frac{c}{2L}$$

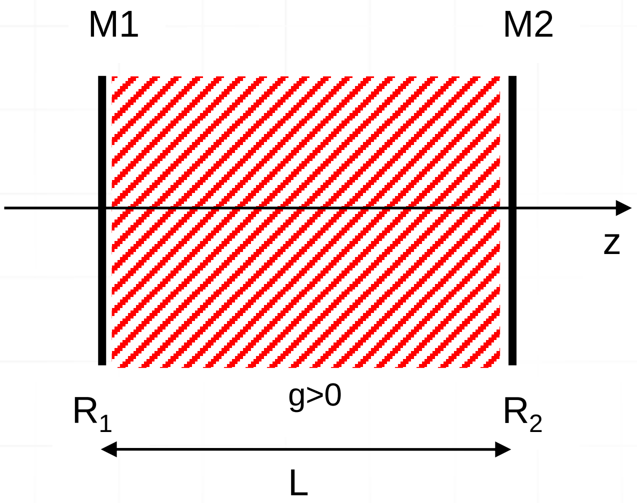
2) Empty cavity linewidth $$\Delta\nu_c = \frac{c}{2L}\frac{1-\sqrt{R_1 R_2}}{\pi\left(R_1 R_2\right)^{1/4}}$$

3) Q-factor $$Q = \frac{\nu_0}{\Delta\nu_c}$$

4) Finesse $$F = \frac{\Delta\nu_{FSR}}{\Delta\nu_c}$$

Laser: resonator + gain

Laser: optical resonator (e.g. a Fabry-Perot) with gain medium inside

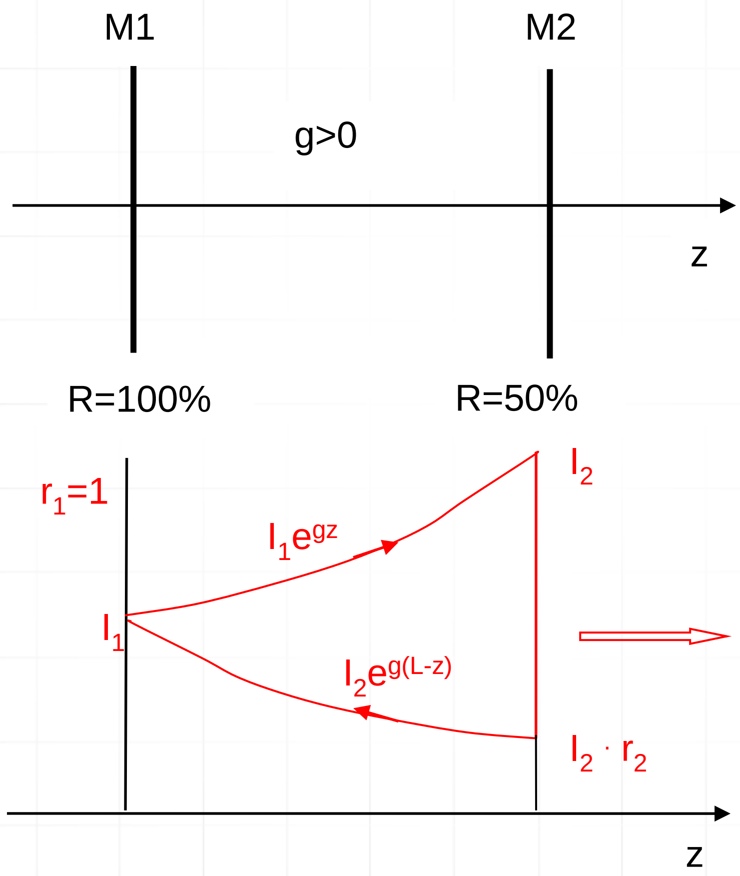


The gain between R_1 and R_2 for the wave propagating from R_1 to R_2 and vice versa is:

$$G_1 = e^{gL} \quad , \quad G_2 = e^{gL} \quad , \quad \text{respectively}$$

So, the round-trip gain is $G_1 G_2$

Resonator with gain medium



Schawlow-Townes equation

$$\Delta \nu_{osc} = 2\pi \frac{(\Delta \nu_c)^2 \cdot h\nu_0}{P_{out}}$$

P_{out} is the output power of the system, $\Delta\nu_c$ is the empty cavity linewidth

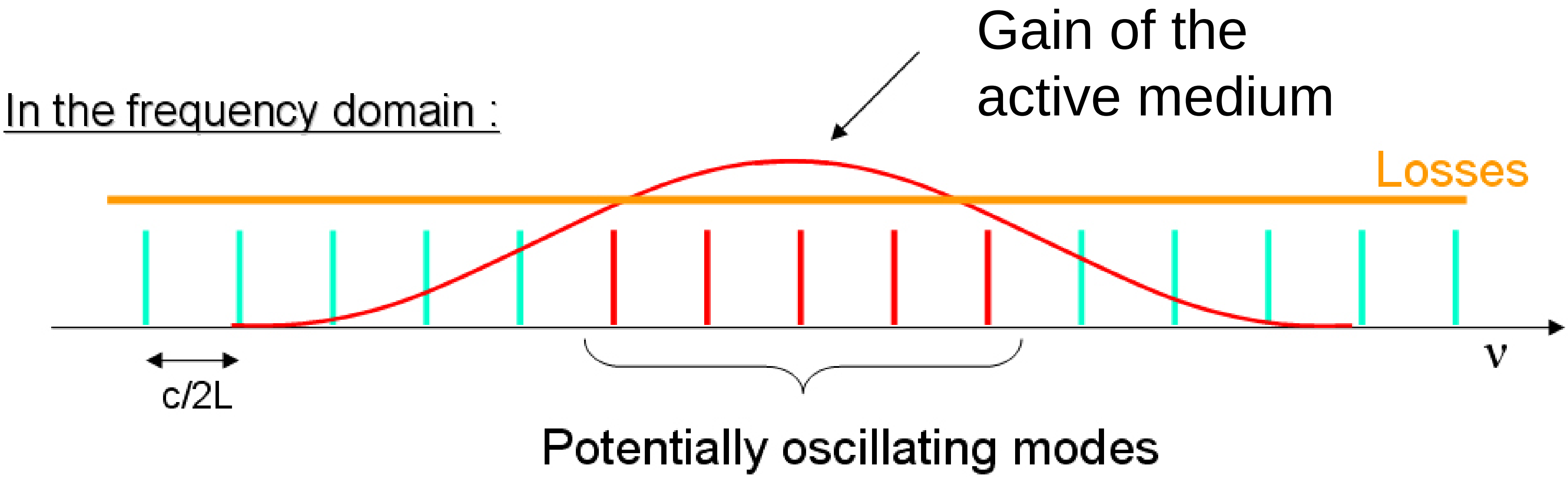
CharlesTownes

Nobel Prize
in Physics
in 1964

Arthur Schawlow

Nobel Prize
in Physics
in 1981

Multimode laser operation



Important

- Population inversion
- Basic parameters of resonators (equations!)
- Draw the transmittance/reflectance of a Fabry-Perot resonator
- Schawlow-Townes equation + description