Lecture 2

Spectrum of a monochromatic plane wave

Let's assume a plane wave, z=const, ϕ = 0

$$\vec{E}(z,t)=\vec{E_0}\exp[i(\omega_0 t-kz)].$$

In order to calculate the spectrum of such wave, we introduce the Fourier transformation:

Fourier transform:

$$X(\omega)=\int_{-\infty}^{+\infty} x(t)e^{(-i\omega t)}dt$$

Inverse Fourier transform:

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{i\omega t}d\omega$$

Transformation of the monochromatic plane wave:

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}[2\pi E_0\delta(\omega-\omega_0)\exp(i\omega t)d\omega]=E_0\exp(i\omega_0 t)$$

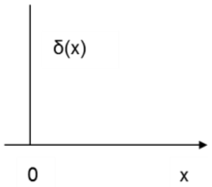
We introduce here δ –Dirac function which has two properties:

(1) $$\delta(\upsilon-\upsilon_0)=\begin{cases}0 & \upsilon\neq\upsilon_0\\ \infty & \upsilon=\upsilon_0\end{cases}$$

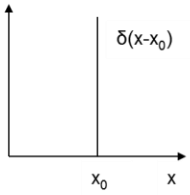
(2) $$\int_{-\infty}^{+\infty}\delta(\upsilon-\upsilon_0)f(\upsilon)d\upsilon=f(\upsilon_0)$$

according to the definition of δ –Dirac function:

$$\int_{-\infty}^{+\infty} \delta(x) f(x)dx = f(0)$$



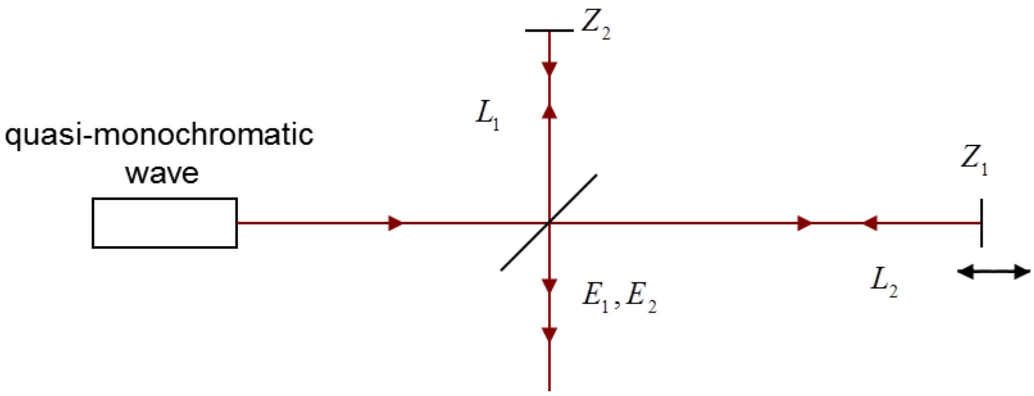
$$\int \delta(x)dx = 1$$



Coherence of electromagnetic waves

The coherence is the ability of the wave to interference with itself.

Michelson interferometer



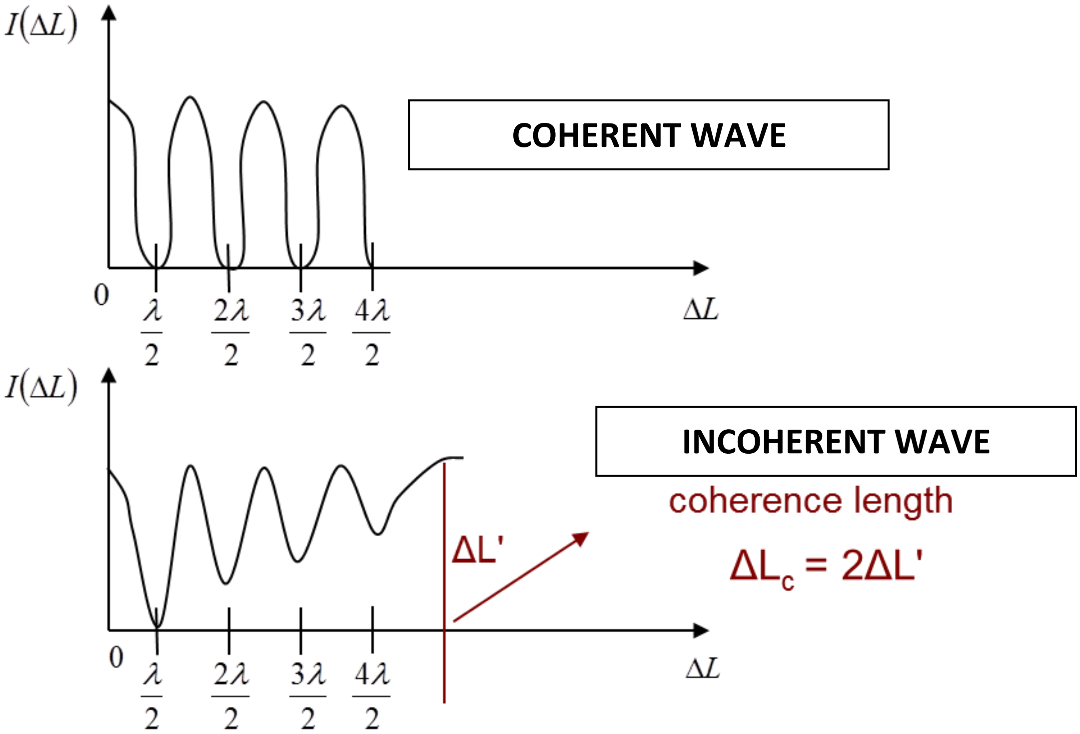
The interference equation:

$$
\begin{aligned}
I = \vec{E}\cdot\vec{E}^{*} &= \left(\vec{E_1}+\vec{E_2}\right)\left(\vec{E_1}+\vec{E_2}\right)^{*} \\
&= \overrightarrow{E_1}\,\overrightarrow{E_1}^{*} + \overrightarrow{E_2}\,\overrightarrow{E_2}^{*} + \overrightarrow{E_1}\,\overrightarrow{E_2}^{*} + \overrightarrow{E_1}^{*}\,\overrightarrow{E_2} = \\
&= \overrightarrow{E_{01}}\exp\left(i(\omega t+\varphi_1)\right)\overrightarrow{E_{01}}\exp\left(i(-\omega t+\varphi_1)\right) + E_{02}\exp\left(i(\omega t+\varphi_2)\right)E_{02}\exp\left(-i(\omega t+\varphi_2)\right) \\
&+ E_{01}\exp\left(i(\omega t+\varphi_1)\right)E_{02}\exp\left(i(-\omega t+\varphi_2)\right) + E_{01}\exp\left(-i(\omega t+\varphi_1)\right)E_{02}\exp\left(i(\omega t+\varphi_2)\right) \\
&= E_{01}^2 + E_{02}^2 + E_{01}E_{02}\exp\left(i(\varphi_1-\varphi_2)\right) + E_{01}E_{02}\exp\left(-i(\varphi_1-\varphi_2)\right) = \\
&= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\,\frac{\exp\left(i(\Delta\varphi)\right)+\exp\left(-i\Delta\varphi\right)}{2}
\end{aligned}
$$

$$I = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos\left(\Delta\phi\right) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\Delta\phi\right)$$

Coherence length and time of coherence:

$$\Delta L_c = c \cdot \Delta t = c \frac{1}{\Delta \upsilon}$$



Wavelength-frequency conversion

The spectral bandwidth can be in optics given in the wavelength domain or in the frequency domain. It is worth to remember, how easy we can transform data between these two domains.

$$\upsilon = \frac{c}{\lambda}$$

after differentiation

$$d\upsilon = -\frac{c}{\lambda^2} \cdot d\lambda$$

Hence

$$\Delta \upsilon = \frac{c}{\lambda^2} \Delta \lambda$$

ν – central frequency of the wave
λ – central wavelength

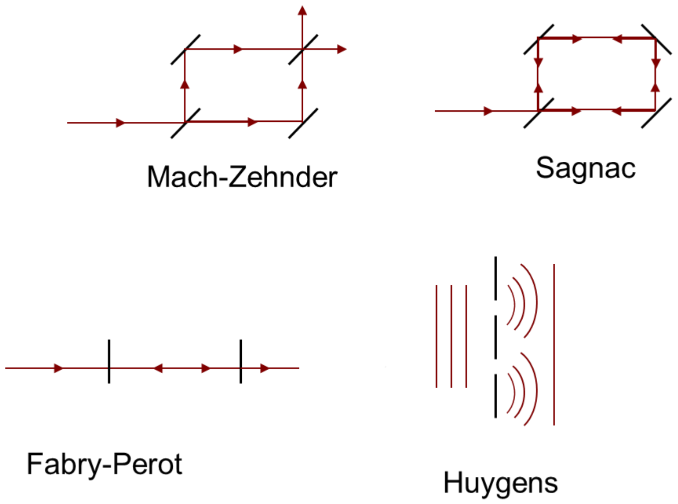
c – velocity of light
Δλ – bandwidth in wavelengths [nm]
Δν – bandwidth in frequency [Hz]

Examples:
Laser diode with central wavelength of λ = 980 nm and bandwidth of Δλ = 1 nm
Δν = 312 GHz , coherence length ΔL = 1 mm

Telecom laser diode: 1550 nm, Δν = 10 GHz → Δλ = 0.08 nm
coherence length ΔL = 3 cm

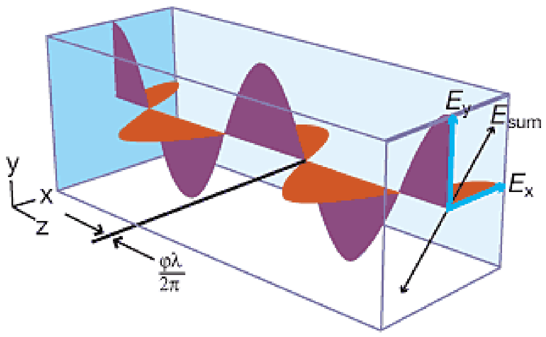
He-Ne laser:
λ = 632 nm, Δν = 30 kHz → Δλ = 4 $\times 10^{-8}$ nm
coherence length ΔL = c/30 kHz = 10 km.

Examples of other interferometers

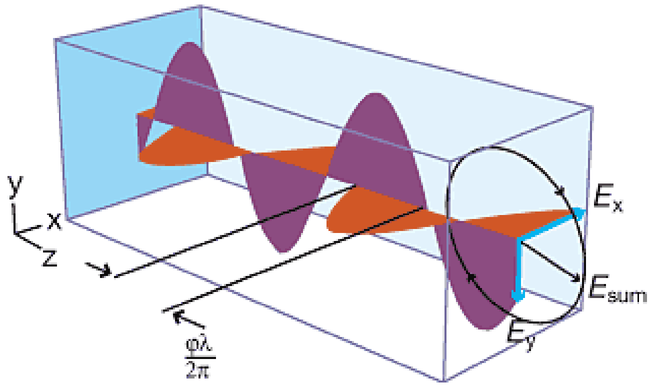


Polarization of waves

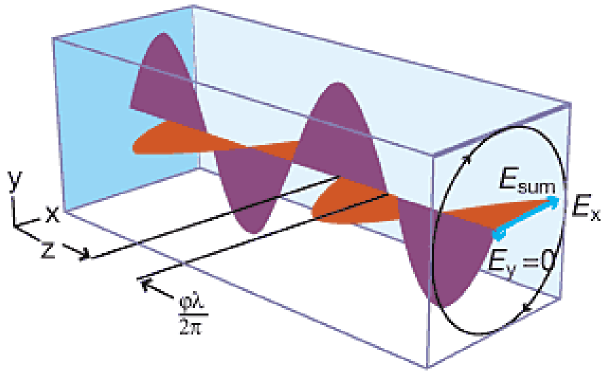
Linear polarization



Elliptical polarization



Circular polarization



Depending on the phase shift between two components:

$$\delta = \frac{\pi}{2} \qquad \text{(RCP - Right Circularly Polarized)}$$

$$\delta = -\frac{\pi}{2} \qquad \text{(LCP - Left Circularly Polarized)}$$

Important:

- **Dirac function – spectrum**
- **Michelson interferometer – principle**
- **Other interferometers (please draw them)**
- **Polarization of waves : types of polarizations**
- **Calculation: bandwidth conversion between frequency and wavelength + calculation of the coherence length**