Lecture 1

Maxwell's equations

	Law	differential form	integral form
1	Gauss for electric fields	$\nabla\vec{D}=\rho$	$\oint_S \vec{D}d\vec{s}=\int_V \rho dv = Q_v$
2	Gauss for magnetic fields	$\nabla\vec{B}=0$	$\oint_S \vec{B}d\vec{s}=0$
3	Faraday	$\nabla\times\vec{E}=-\frac{d\vec{B}}{dt}$	$\oint_L \vec{E}d\vec{l}=-\int_S \frac{d\vec{B}}{dt}\cdot d\vec{s}=-\frac{d\Psi_B}{dt}$
4	Ampere	$\nabla\times\vec{H}=\vec{j}+\frac{d\vec{D}}{dt}$	$\oint_L \vec{H}d\vec{l}=\int_S\left(\vec{j}+\frac{d\vec{D}}{dt}\right)\cdot d\vec{s}=I+\frac{d\Psi_D}{dt}$
	Sometimes so called 5th M. equation	$\nabla\vec{j}=-\frac{d\vec{\rho}}{dt}$	$\oint_S \vec{j}d\vec{s}=I-\frac{dQ_v}{dt}$

$\vec{D}-electric\ displacement\ field\quad or\quad electric\ flux\ density\ \left[\frac{C}{m^2}\right]$

$\rho - charge\ density\ \left[\frac{C}{m^3}\right]$

$Q_v - total\ charge\ in\ V\ volume\ [C]$

$\vec{B}-magnetic\ flux\ density\ \left[\frac{vs}{m^2}\right]=\left[\frac{Wb}{m^2}\right]$

$t - time\ [s]$

$\vec{E}-electric\ field\ \left[\frac{V}{m}\right]$

$\Psi_B - magnetic\ flux\ [Wb]$

$\vec{H}-magnetic\ field\ \left[\frac{A}{m}\right]$

$\vec{j}-current\ density\ \left[\frac{A}{m^2}\right]$

$I - current\ [A]$

Materials equations

(1)
$$\vec{D} = \varepsilon \vec{E}$$
$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$
$$\vec{D} = \varepsilon_0 \vec{E} + \varepsilon \cdot \chi \vec{E}$$
$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$$
$$\varepsilon = \varepsilon_0 \varepsilon_r$$

ε – *permittivity*
ε_0 – *permittivity of vacuum*
$\varepsilon_0 = 8.854 \cdot 10^{-12} \left[\frac{F}{m}\right]$
$\dot{P}$ – *electric polarization*
χ – *electric susceptibility*
ε_r – *relative permittivity*

(2)
$$\vec{B} = \mu \cdot \vec{H}$$

μ – *permeability*
μ_0 – *vacum permeability*
$\mu_0 = 4\pi \cdot 10^{-7} \left[\frac{H}{m}\right]$

(3)
$$\vec{j} = \sigma \vec{E}$$

σ – *electrical conductivity of the medium* $[\Omega \cdot m]^{-1}$ 7

The wave equation

We assume a medium without charge (ρ = 0). From the Faraday's and Ampere's law:

$$\begin{cases} \nabla \times \vec{E} = -\dfrac{d\vec{B}}{dt} = -\mu \dfrac{d\vec{H}}{dt} \\ \nabla \times \vec{H} = \dfrac{d\vec{D}}{dt} + \sigma \vec{E} = \varepsilon \dfrac{d\vec{E}}{dt} + \sigma \vec{E} \end{cases}$$

Taking curl (rotation) of the first equation (both sides)

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\mu \frac{d\left(\nabla \times \vec{H}\right)}{dt}$$

We can rewrite the left side of the equation by using the so called triple vector product:

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{b}\left(\vec{a} \cdot \vec{c}\right) - \vec{c}\left(\vec{a} \cdot \vec{b}\right)$$

and we obtain:

$$\nabla \times \left(\nabla \times \vec{E}\right) \equiv \nabla\left(\nabla \vec{E}\right) - \nabla^2 \vec{E}$$

we assumed $\rho = 0$ $\left(\nabla \vec{D} = \rho = 0\right)$, therefore $\nabla \vec{E} = 0$

Hence we obtain:

$$\nabla^2 \vec{E} = \mu\varepsilon \frac{d^2 \vec{E}}{dt^2} + \mu\sigma \frac{d\vec{E}}{dt}$$

After elementary generalization – **<u>the wave equation</u>**

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{d^2 \vec{E}}{dt} - \mu\sigma \frac{d\vec{E}}{dt} = 0$$

When the medium is nonconductive, σ = 0, the wave equation for the free space:

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{d^2 \vec{E}}{dt^2} = 0$$

Solution of the wave equation: **plane wave** (here written with no attenuation)

$$\vec{E}(t, \vec{r}) = \vec{E}_0 \exp\left(i\omega t - i\vec{k}\vec{r}\right)$$

Important:

1) **Maxwell's equations**
2) **Derivation of the wave equation from Maxwell's equations**